Dynamic Instability of a Cable in Incompressible Flow

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Theme

THE dynamic stability of a cable stretched between two supports is analyzed. The cable is exposed to a uniform, incompressible flow at an angle to the cable axis. Fluid forces are taken to be the quasi-static components of pressure drag and friction drag. Response due to vortex shedding is not considered. A new instability is found which is wavelike in character and has dominant amplitude at the rear support. The instability is due to the presence of bending stiffness in the cable, an effect previously thought negligible and not before considered. The results are important conceptually for studies of cables exposed to flow.

Contents

Consider an elastic cable suspended between two points A and B (Fig. 1). The cable is of circular cross section, uniform diameter and mechanical properties along its length and is subjected to an initial tensile load T. It is assumed that deflections are small and that tension is constant along the cable as well as constant in time. Bending stiffness of the cable is included. The problem is two dimensional, i.e., the flow vector, the gravitational vector and the cable all lie in a vertical plane. The flow vector is perpendicular to the gravitational vector, as in the case of an airplane towing a drogue.

Aerodynamic forces on the cable are found using a cross-flow concept known to be valid for Reynolds numbers between 10^2 and 10^5. The flow vector is resolved into components parallel and perpendicular to the cable. Force components are calculated from the instantaneous components of velocity. This quasi-static approach is most accurate at low frequencies of motion.

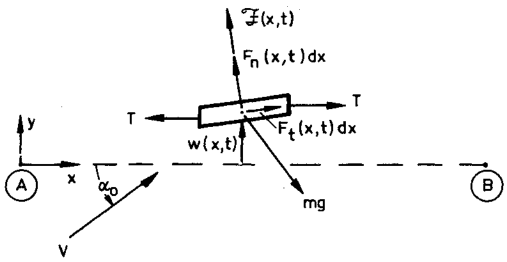

Fig. 1 Cable suspended between points A and B. Displaced position; not to scale. Bending and shearing forces not shown.

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$$F_n = \tfrac{1}{2} C_d \rho V_n^2 D \tag{1}$$

$$F_t = \tfrac{1}{2} C_f \rho V_t^2 \pi D \tag{2}$$

The coefficients of drag and friction are C_d and C_f respectively, ρ is the fluid density, V_n and V_t are the normal and tangential velocity components. and D is the cable diameter. If the initial angle between cable and flow is nonzero, the small deflection assumption provides $w'(x,t) \ll \alpha_0$. Following Phillips,[1] trigonometric terms in the velocity components are expanded and terms of order w^2 neglected

$$F_n = C_d D q \{\sin^2 \alpha_0 - w'(x,t)\sin 2\alpha_0 - (2/V)\dot{w}(x,t)\sin \alpha_0\} = 0 \tag{3}$$

$$F_t = C_f \pi D q \{\cos^2 \alpha_0 + w'(x,t)\sin 2\alpha_0\} \tag{4}$$

Euler-Bernoulli beam theory now provides the equations of motion. For the sake of generality, the equations are first written with variable tension

$$[EIw''(x,t)]'' - [T(x,t)w'(x,t)]' + [C_d qD \sin 2\alpha_0 - C_f q\pi D \cos^2 \alpha_0]w'(x,t) + C_d D\rho V(\sin \alpha_0)\dot{w}(x,t) + m\ddot{w}(x,t) = \mathscr{F}(x,t) + C_d qD \sin^2 \alpha_0 - mg \cos \alpha_0 \tag{5}$$

$$T'(x,t) + [C_f q\pi D \sin 2\alpha_0 - C_d qD \sin^2 \alpha_0]w'(x,t) - m\ddot{u}(x,t) = -C_f q\pi D \cos^2 \alpha_0 - mg \sin \alpha_0 \tag{6}$$

where m is the mass per unit length of the cable and $\mathscr{F}(x,t)$ is the unsteady force due to vortex shedding. Equation (6) is useful when one wishes to calculate the actual variation of tension due to gravitational and frictional forces; however, we hypothesize T to be constant from this point on.

For the stability study, the deflection is broken into static and dynamic components

$$w(x,t) = w_s(x) + w_d(x,t) \tag{7}$$

The constant tension assumption then uncouples the equation for $w(x,t)$ from that for $u(x,t)$

$$\{EIw_s^{IV}(x) - Tw_s''(x) + [C_d qD \sin 2\alpha_0 - \pi C_f qD \cos^2 \alpha_0]w_s'(x) - C_d qD \sin^2 \alpha_0 + mg \cos \alpha_0\} + \{EIw_d^{IV}(x,t) - Tw_d''(x,t) + [C_d qD \sin 2\alpha_0 - \pi C_f qD \cos^2 \alpha_0]w_d'(x,t) + C_d \rho VD(\sin \alpha_0)\dot{w}_d(x,t) + m\ddot{w}_d(x,t)\} = \mathscr{F}(x,t) \tag{8}$$

The first term in braces is constant in time and must independently vanish, thereby yielding the static deflection. The dynamic deflection is found from the remaining portion of the equation. Furthermore, the equation is linear and the particular solution corresponding to the random function $\mathscr{F}(x,t)$ can be found separately. The random response is not of interest here and will not be studied.

The stability of the system is determined by equating the second term in braces in Eq. (8) to zero. Nondimensional variables are first introduced

$$\xi = x/L \tag{9}$$

$$\tau = (T^2/mEI)^{1/2} t \tag{10}$$

$$y(\xi, \tau) = w_d/D \tag{11}$$

The stability equation becomes

$$\varepsilon^4 y^{IV}(\xi,\tau) - \varepsilon^2 y''(\xi,\tau) + \delta\varepsilon y'(\xi,\tau) + \gamma\dot{y}(\xi,\tau) + \ddot{y}(\xi,\tau) = 0 \tag{12}$$

where the ratio of bending to stretching forces is

$$\varepsilon = (EI/TL^2)^{1/2} \tag{13}$$

A ratio of aerodynamic and bending forces to stretching forces is

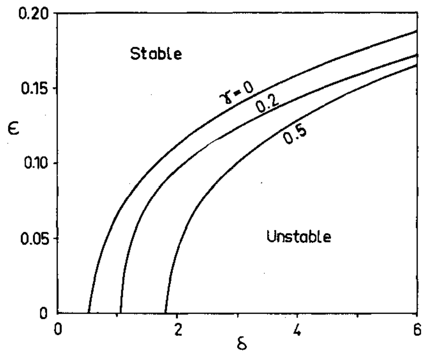

Fig. 2 Stability boundary for a clamped-clamped cable.

$$\delta = (C_d qD \sin 2\alpha_0 - \pi C_f qD \cos^2 \alpha_0)(EI/T^3)^{1/2} \quad (14)$$

and a measure of aerodynamic damping is

$$\gamma = C_d D\rho V \sin \alpha_0 (EI/mT^2)^{1/2} \quad (15)$$

Boundary conditions for either the pinned or clamped ends are now applied. The clamped case is, for example,

$$y(0, \tau) = y(1, \tau) = y'(0, \tau) = y'(1, \tau) = 0 \quad (16)$$

When a separated solution of the form $y(\xi, \tau) = Y(\xi)e^{k\tau}$ is assumed, an eigenvalue problem results. This problem has been studied at length in the field of panel flutter. Important contributions have been made for small values of ε by Spriggs, Messiter and Anderson,[2] Dugundji,[3] Erickson and others. Spriggs' paper gives a singular perturbation solution for small ε, yielding characteristic lengths and periods of motion. Dugundji gives extensive numerical results.

Solutions have time dependence of the form $e^{k\tau}$, where k is the complex frequency. One studies the complex frequency as a function of the parameters ε, δ, and γ. When the real part of k becomes positive, the motion grows in time. The corresponding flutter mode has the largest amplitude at the rear support, in an edge layer of characteristic length $(EI/T)^{1/2}$. The characteristic period of motion is $(mEI/T^2)^{1/2}$. For mode shapes, the reader is referred to Dugundji's study of supersonic panel flutter.[3]

One can define a flutter surface in the (ε, δ, γ) space on which the real part of k changes sign (Fig. 2). The case of zero bending stiffness lies at the origin, in the stable region. Theories which have neglected bending stiffness therefore could not predict the instability found here. If the dynamic pressure or the length of the cable is increased, this has a destabilizing effect on the cable. On the other hand, increasing the tension or the aerodynamic damping (for instance through the mass ratio) is stabilizing. The role played by bending stiffness is unusual because the locus of increasing bending stiffness is a straight line radiating from the origin. Hence, for a very flexible cable which is initially stable, increasing the bending stiffness may cause an instability.

A conservative, but useful design criterion is given by the intersection of the zero damping curve with the δ axis. If $\delta < 0.5443$, one can be sure that flutter will not occur. (This corresponds to the zero order singular perturbation solution for no damping.)

Numerical studies indicate that the flutter can occur at moderate diameters, lengths and velocities, but that the instability is overcome by tension on the order of the weight of the cable. This means that the instability is likely to become a practical problem only in special flow situations or where transient loading situations reduce the tension for several times the characteristic period of motion.

An extension of this work can be made to the case of cables with ends rigidly fixed in space. The tension in the cable then grows with static lateral deflection. This is a nonlinear structural problem; nevertheless, the tension in the cable can be found as a first step and then the same flutter curves applied. This case is largely self-stabilizing because of the growth of tension with increasing dynamic pressure. Civil structures using cables anchored at both ends are not likely to see the instability discussed here.

In conclusion, then, a new dynamic instability has been predicted for a cable suspended between two supports. Incompressible fluid flow over the cable can cause a fluttering motion with dominant amplitude at the downstream support. This is a new instability, not related to transmission line galloping or other vortex shedding problems. It is due to the quasi-static drag on the cable, and is related to the instability predicted by Phillips in 1948 for an infinitely long cable. The instability is important from a conceptual standpoint, because it points out the effect of bending stiffness near a support. The presence of even an infinitesimal amount of bending stiffness provides the mechanism for flutter. Perhaps bending stiffness will play a similar role in other cable problems, and may trigger instabilities in systems such as three-dimensional rotating systems. If unexplainable instabilities occur in cable experiments, it would be advisable to analyze the system carrying bending stiffness terms in the region of cable supports, perhaps for a distance of 10 times the characteristic edge layer length.

The weakness of the present theory lies in the small deflection and constant tension assumptions. These conditions do not hold often in practice. Nevertheless, the current results are valid qualitatively in general and quantitatively for many special cases. The present study is valid for cables with pinned or clamped ends but has not been extended to the free-end case.

References

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[2] Spriggs, J. H., Messiter, A. F., and Anderson, W. J., "Membrane Flutter Paradox—an Explanation by Singular-Perturbation Methods," *AIAA Journal*, Vol. 7, No. 9, Sept. 1969, pp. 1704–1709.

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